

defining n and ρ , the equation reads:

$$\frac{d^2 R}{d\rho^2} + \frac{2}{\rho} \frac{dR}{d\rho} - \left\{ \frac{1}{4} - \frac{n}{\rho} + \frac{l(l+1)}{\rho^2} \right\} R = 0. \quad (3)$$

Substituting

$$R = e^{-1/2\rho} \rho^l F(\rho), \quad (4)$$

we get the equation:

$$\rho \frac{d^2 F}{d\rho^2} + (2l + 2 - \rho) \frac{dF}{d\rho} + (n - 1 - l) F = 0 \quad (5)$$

for the confluent hypergeometric function *)

$$F = F(l + 1 - n, 2l + 2, \rho). \quad (6)$$

When n is an integer the series expansion for F breaks off so as to give derivatives of Laguerre polynomials. The wave function then has a node at $r_0 = \infty$, being the normal boundary condition of the free hydrogen atom.

The purpose of this article is to calculate how the 1s, 2s and 2p levels of the hydrogen atom are changed when it is uniformly compressed, i.e. when a node lies at finite r_0 . The energy levels are shifted to higher values through the influence of the potential wall. Corresponding values of energy E and radius r_0 of the cage will be calculated for the whole range beginning with the large values of r_0 . The notation †) "1s, 2s and 2p level" will be maintained, although the number n is only an integer for $r_0 = \infty$. The quantum number l is of course not changed by compression.

§ 2. *The method of Michels, De Boer and Bijl.* For a radius r_0 so large, that the deviation of E from its value at $r_0 = \infty$ is still very small, Michels, De Boer and Bijl¹⁾ have developed an approximative method. They calculated the shift

$$*) \quad F(z, \gamma, \rho) = 1 + \frac{\alpha}{\gamma} \rho + \frac{\alpha(\alpha+1)}{\gamma(\gamma+1)} \frac{\rho^2}{2!} + \dots \quad (7)$$

The confluent hypergeometric functions have specially been investigated by Whittaker²⁾. The connection of his symbols k , m and z with the variables l , n , and ρ used here is: $k = n$, $m = l + 1/2$ and $z = \rho$. Buchholz³⁾ parameters $\nu \equiv i\tau$, p and $z \equiv i\zeta$ are $\nu = n$, $p = 2l + 1$ and $z = \rho$.

†) Indicating the first symbol of this notation by N and the second by l , the number N can be defined by saying that the wave function " Nl " has $N - l - 1$ nodes between its limiting points. This "principal quantum number" N coincides with the variable n (corresponding with E by formula (2)) for $r_0 = \infty$ only.